Representing Multiplication with Arrays

Representing mathematical relationships is a key element of developing mathematical understanding. In Grade 3, students first encounter multiplication as "groups of" as they brainstorm lists of things that come in equal groups of various sizes and create multiplication situations from those lists. These contexts help them develop visual representations which give meaning to multiplication expressions. For example, 5×4 can be visualized as 5 dogs with 4 legs each or 5 rectangles with 4 sides on each rectangle.

Students extend their understanding of multiplication through their work with arrays. For multiplication, the rectangular array is an important tool: it highlights key properties of multiplication, provides a tool for solving problems, and can be extended as students apply ideas about multiplication to new kinds of numbers (e.g., fractions) in later grades.

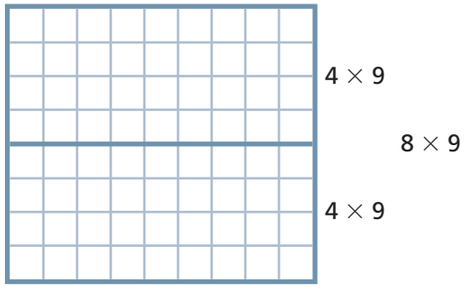
Learning to Use Arrays

Through using rectangular arrays, students come to see how a rectangle can be composed of square units. At first they may count all the squares or add the number in each row to find the total number of squares. Over time, they develop a mental image of the structure of the array as an arrangement of rows (or columns) with an equal number of squares in each. They come to see how this structure represents multiplication, with one factor represented by the number of rows and the other factor represented by the equal number of square units in each row. While the way in which an array models multiplication may be obvious to adults, students need time and experience before they fully grasp how to see the multiplication in a rectangular array. It is useful to focus students on this idea by asking questions such as, "Where do you see the multiplication, 12×4, in this array?"

Why Rectangular Arrays for Multiplication?

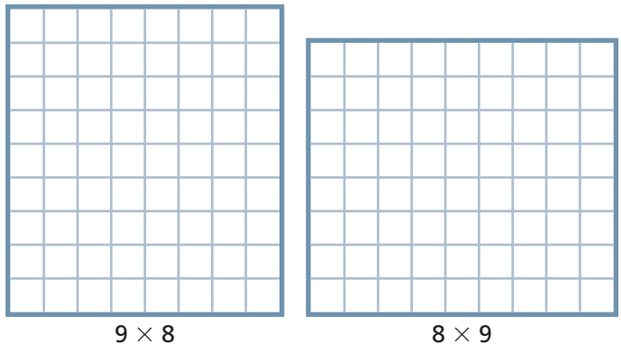
As students come to understand the operation of multiplication in Grades 3 and 4, they gradually move away from thinking of multiplication only as repeated addition. They learn that multiplication has particular properties that distinguish it from addition. Although a number line or 100 chart can show how multiplication can be viewed as adding equal groups, neither of these tools provides easy access to other important properties of multiplication. The rectangular array provides a window into properties that are central to students' work in learning the multiplication facts and in solving multidigit multiplication and division problems. Arrays are also a tool for understanding area, which students encounter in Unit 4.

For example, suppose that a student is reviewing one of the more difficult multiplication facts, 8×9. The student might think of splitting this multiplication in this way: $8 \times 9 = (4 \times 9) + (4 \times 9)$, using a strategy based on the distributive property of multiplication, which can be represented by using an array.

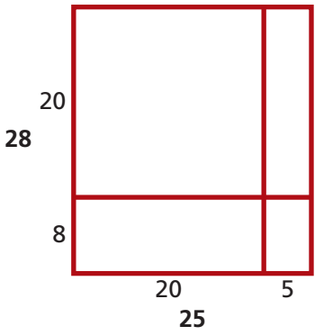


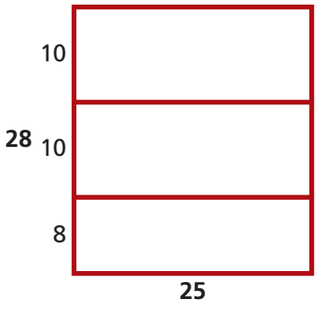
One factor is broken up into parts. Each part must be multiplied by the other factor. The array shows how one dimension of the rectangle (8) is split into parts $(4 + 4)$ to form two new rectangles, each with the dimensions 4×9. This property of multiplication is at the core of almost all common strategies used to solve multiplication problems.

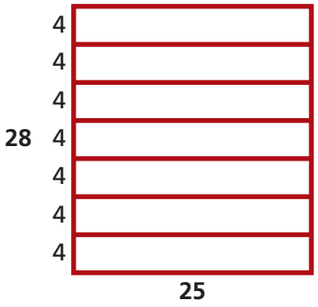
The rectangular array is also a tool to help students visualize why the product of 9×8 is the same as the product of 8×9. The array can be rotated to show that nine rows with eight in each row has the same total number of squares as eight rows with nine in each row. The column on one becomes the row on the other, illustrating the commutative property— the fact that when you change the order of two factors in a multiplication expression, the product remains the same.



Arrays are particularly useful for solving or visualizing how to solve multidigit multiplication problems. After students have worked with rectangular arrays for single-digit multiplication facts and thoroughly understand how an array represents the factors and product, they can use arrays in their work to solve harder problems. The array for 28 × 25 can be broken up in many ways.

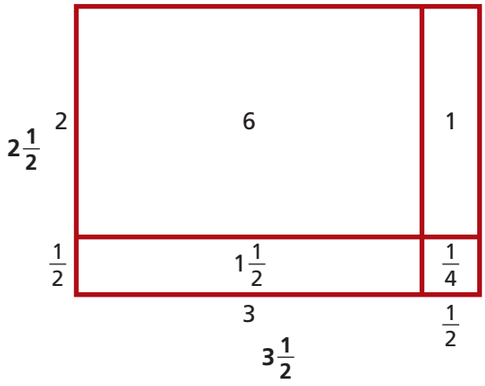


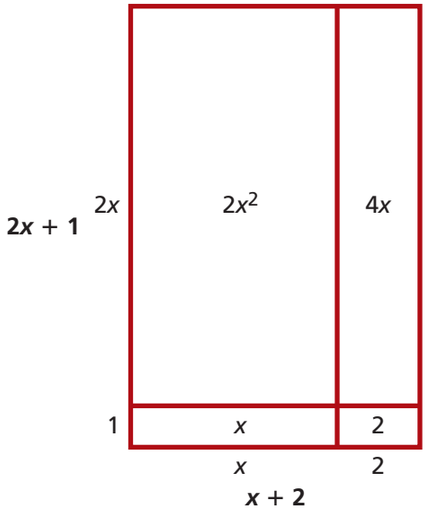




Each of these ways suggests an approach to solving this problem.

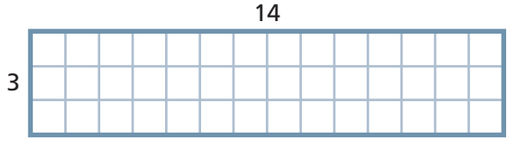
Finally, the use of the rectangular array can be extended in later grades as students work with multiplication of fractions and, later, of algebraic expressions.





Labeling Arrays

For multiplication notation to describe arrays, the *Investigations* curriculum uses the convention of designating the number of rows first and the number in each row second; for example, 3 × 14 indicates 3 rows with 14 in each row.



This convention is consistent with using 3 × 14 to indicate 3 groups of 14 in other multiplication situations (e.g., 3 bags with 14 marbles in each bag). However, it is not necessary or useful to spend time getting students to follow this system rigidly; trying to remember which number stands for rows and which for the number in a row can be unnecessarily distracting for students. When students suggest a multiplication expression for an array, what is important is that they understand what each number means and that they can explain how it is represented by an array. For example, a student might write either 3 × 14 or 14 × 3 to label an array that represents 3 rows of cans with 14 in each row.

Note that in other cultures, conventions about interpreting multiplication expressions differ. In some countries, the convention for interpreting 3 × 14 is not "3 groups of 14" but "3 taken 14 times."